Earth, and from various Bodies placed on, or near the surface of the Earth." By James Glaisher, Esq. Communicated by G. B. Airy, Esq., F.R.S., Astronomer Royal, &c.

The author enters into a very detailed description of the construction of the thermometers he employed in these observations, and the precautions he took to ensure their accuracy; and gives tabular records of an extensive series of observations, amounting to a number considerably above ten thousand, with thermometers placed on nearly a hundred different substances, exposed to the open air, under different circumstances, and in various states of the sky, at the Royal Observatory at Greenwich.

February 18, 1847.

The MARQUIS OF NORTHAMPTON, President, in the Chair.

Edward John Rudge, Esq., was elected a Fellow of the Society.

"On the Diurnal Variation of the Magnetic Declination of St. Helena." By Lieut.-Colonel Edward Sabine, R.A., For. Sec. R.S.

It has long been known that the diurnal variation of the magnetic needle is in an opposite direction in the southern, to what it is in the northern hemisphere; and it was therefore proposed as a problem by Arago, Humboldt and others, to determine whether there exists any intermediate line of stations on the earth where those diurnal variations disappear. The results recorded in the present paper are founded on observations made at St. Helena during the five consecutive years, from 1841 to 1845 inclusive; and also on similar observations made at Singapore, in the years 1841 and 1842; and show that at these stations, which are intermediate between the northern and southern magnetic hemispheres, the diurnal variations still take place; but those peculiar to each hemisphere prevail at opposite seasons of the year, apparently in accordance with the position of the sun with relation to the earth's equator.

February 25, 1847.

The MARQUIS OF NORTHAMPTON, President, in the Chair.

The Earl of Hardwicke was elected a Fellow of the Society.

Rev. J. O. W. Haweis, M.A., was put to the ballot, but not elected.

"On certain Properties of Prime Numbers." By the Right Hon. Sir Frederick Pollock, M.A., F.R.S., Lord Chief Baron of the Exchequer, &c.

The author of this paper, after noticing Wilson's Theorem, (published by Waring about the year 1770, without any proof), which theorem is that, if A be a prime number, 1. 2. 3. . . . (A—1)+1 is divisible by A; refers to Lagrange's and Euler's demonstrations, and mentions Gauss's extension of the theorem, to any number, not prime; provided that instead of 1, 2, 3, &c. (A—1), those numbers only be taken which are prime to A, and 1 be either added or subtracted. This theorem was published by Gauss without a proof in 1801, with a rule as to the cases in which 1 is to be added or subtracted, the correctness of which is questioned by the author, who proceeds to propound the following theorem, which he had previously, for distinctness, divided into three.

If any number, prime or not, be taken, and the numbers prime to it, and less than one half of it be ascertained, and those be rejected whose squares ± 1 are equal to the prime number, or some multiple of it (which may be more than one), then the product of the remaining primes (if any), ± 1 shall be divisible by the prime number.

He gives as examples, 14, the primes to which, and less than one half, are 1, 3, 5, and $1.3.5=15$; therefore $1.3.5-1=14$; also 15, the primes to which and less, are 1, 2, 4, 7; but $4\times 4=16=15+1$; therefore 4 is to be rejected, and $1.2.7+1=15$. The author adds another theorem, that if A be a prime number, all the odd numbers less than it (rejecting as before); also, all the even numbers (making the same rejection except A—1) will, multiplied together, be equal to $A\pm 1$.

The author then proceeds to prove Gauss's extension of Wilson's theorem, and to give the cases in which 1 is to be added or subtracted; and in the course of the proof, he mentions that the numbers prime to any number not only are found in pairs, one greater and one less than one-half of the number, but that they associate themselves in sets of four, with an odd pair in certain cases. Thus, the primes to 7 are 1, 2, 3, 4, 5, 6,—

$$2\times 4=8=7+1.$$

Put the complemental numbers underneath crosswise, thus,—

$$\begin{array}{ccc} 2 & \times & 4 \\ 3 & \times & 5 \end{array}$$

so that $2+5$ and $4+3$ may equal 7; and then

$$\begin{array}{ll} 3\times 5=15=2\times 7+1 & \\ 2\times 3=\ 6=7-1 & \qquad 4\times 5=20=3\times 7-1 \end{array}$$

Multiplied together one way the product exceeds 7, or a multiple of it, by 1; multiplied the other way, the product is less than 7, or some multiple of it, by 1. By assuming the prime number to be A, and the two primes to it to be p, q, and that $p+q$ be not equal to A, but $pq=nA\pm 1$, it is shown that the complemental primes

(A$-q$) and (A$-p$) will have a product$=n'$A± 1, and that, instead of 1, the number may be any other prime to A. Upon this foundation the author proceeds to show that Wilson's theorem, and also Gauss's, may be made much more general; that if A be a prime number, as 7, the numbers less than it may be arranged in pairs, not only with reference to 1, but to any number less than 7. Take 4 as an example:—

$$\begin{array}{ccl} 1 & \times & 3=7-4 \\ 4 & \times & 6=4\times 7-4 \\ 2 & \times & 5=2\times 7-4 \end{array}$$

therefore $1.2.3.4.5.6=7n-4^3$;
therefore $1.2.3.4.5.6+4^3=7n$; that is, is divisible by 7.

The same is then shown as to numbers not prime, provided those numbers alone are taken which are prime to it, and the number of pairs will be half the number of primes. The general theorem therefore is this:—If A be any number, prime or not, and m be the number of primes to it, which are $1, p, q, r$, &c.; then $1.p.q.r$, &c., $\pm Z^{\frac{m}{2}}$ will be divisible by A, provided Z be prime to A, whether it be greater or less.

It follows from this that $z^{\frac{m}{2}}\pm 1$ must be divisible by A, and therefore that z^m-1 must be divisible by A. If A be a prime number and z a number prime to it (which every number not divisible by it is), this is Fermat's theorem, and the author has given a new proof of it. But the theorem is true though A be not a prime number, provided z be prime to A and m be the number of primes to A, and less than it; and instead of 1, any other number prime to A raised to the mth power may be substituted: and z^m-y^m will be divisible by A, provided z and y be primes to A, and m be the number of primes to A and less than it.

The author has therefore in this paper offered a proof of Gauss's theorem, and proved that it applies in certain cases to one half of the primes, and in all cases, with certain modifications, has shown that a similar property belongs to the product of the odd numbers, and also of the even numbers which precede any prime number; and lastly, has shown the intimate connexion between Wilson's theorem and Fermat's, and shown that each is but a part of a much more general proposition, which, he observes, may itself turn out to be part only of a still more universal one.

In a postscript, the author has shown that the well-known law of reciprocity of prime numbers is an immediate corollary from his theorem; and that it may be extended thus: if A and B be any two numbers (not prime numbers but) prime to each other, and the primes to A, and less than it, are (m) in number, and the similar primes to B are (n), then (A^n-1) is divisible by B, and (B^m-1) is divisible by A.